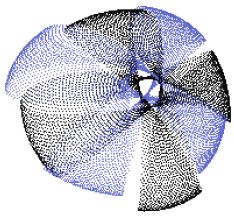
DIOPHANTINE GEOMETRY (MATH 272Z) – SYLLABUS

MAX WEINREICH

Basic information.

- Meets Spring 2026 at Harvard, Monday 10:30 AM – 11:45 AM and Wednesday 10:30 AM – 11:45 AM.
- Our first meeting is 1/26 and last meeting is 4/29. There is no class 2/16, 3/16, 3/18 due to breaks.
- This is a typical graduate topics course, except that some time in class each week will be dedicated to problem solving.
- Anyone taking the class for a grade/credit will complete a written final project, the length to be determined based on number of enrollees. The grade will also incorporate biweekly problem sets in which you can choose interesting problems from a long menu.

Course goals. This course is aimed at first and second year graduate students who want to be able to understand papers and talks in arithmetic geometry. We will focus on using algebraic geometry to gain insight into the sets of solutions of Diophantine equations, or equations in integers. At opportune moments, we'll digress in a dynamical direction, seeing how heights are used by working number theorists and dynamicists to prove finiteness results of various kinds and guess dynamical behavior. We'll also see how to write computer programs to visualize some important phenomena from arithmetic geometry, like the "cronut" below.



Game plan. The content will be roughly organized into three units.

(1) Height theory (January–February). Taking [HS00, Chapter B] as our primary text, we will cover the following topics.
   - Introduction to the field of arithmetic geometry
   - The Weil height on $\mathbb{Q}$, absolute values, and the product formula
   - Heights on number fields and projective spaces
   - Transformation of height under application of a rational map or morphism
   - Northcott's theorem
   - Canonical heights
   - The uniform boundedness conjecture

- Heights on quasi-projective varieties
- The Weil height machine
- Arithmetic degrees of rational maps
- Canonical heights for correspondences

(2) Fundamentals of elliptic curves (March). Following [Sil09, Chapter 3], we will cover at least the following topics:
- Weierstrass equations
- The group law
- Isogenies
- The invariant differential
- Dual isogenies
- The Tate module
- The Weil pairing

(3) Abelian varieties, Jacobians, and rational point theorems (April), mainly following [HS00, Chapter A,C]. We will carefully cover:
- Abelian varieties
- Jacobians of curves
- Mordell-Weil theorem for abelian varieties over number fields

We will discuss without full proof:
- Roth's theorem
- Siegel's theorem on finiteness of integral points
- Falting's theorem on rational points
- Integrable systems

Course policies.

(1) There is no formal requirement for joining our course, but undergraduates and students at other schools interested in the course should email me before the first day of class to make sure they have the necessary background knowledge.
(2) No AI.
(3) Attendance will not be taken, but do come to class. Late work will not be accepted. Assignments that cannot be turned in on time due to extenuating circumstances will be dropped from the grade, except for the final project, which is non-negotiable.
(4) Further information about assignments, office hours, and so on will be posted on our Canvas page starting on the first day of the semester.

References

[HS00] Marc Hindry and Joseph H. Silverman. *Diophantine geometry*, volume 201 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2000. An introduction.

[Sil09] Joseph H. Silverman. *The Arithmetic of Elliptic Curves*, volume 106 of *Graduate Texts in Mathematics*. Springer-Verlag, 2009.